Math 117 - Fall 2022 - Common Final Exam, version A Solutions

1. (6 points) Ten inches of snow is equivalent to one inch of rain. Write an equation for the amount of precipitation, measured in inches of rain, $r = f(s)$ as a function of the equivalent number of inches of snow s.

Solution: $r = f(s) = \dfrac{s}{10}$

2 points	At least some expression giving r as a function of s
1 points	formula on the right track, may have $r = 10s$
3 points	correct formula

2. (8 points) Given the function $f(x) = 2 - x^2$, compute the average rate of change of f between $x = 1$ and $x = 4$. Show your work.

Solution:

$$\frac{\Delta f}{\Delta x} = \frac{f(4) - f(1)}{4 - 1} = \frac{2 - 16 - (2 - 1)}{3} = \frac{-15}{3} = -5$$

2 points	Some quotient, might be $\Delta x / \Delta f$ or have incorrect endpoints
2 points	correct quotient $\Delta f / \Delta x$
2 points	progress toward numerical value, may have errors
2 points	correct numerical value

3. We have \$24 to spend on vegetables and fruit. A pound of vegetables costs \$1 and a pound of fruit costs \$2. The number of pounds of vegetables we can afford, y, is a function of the number of pounds of fruit we buy, x.

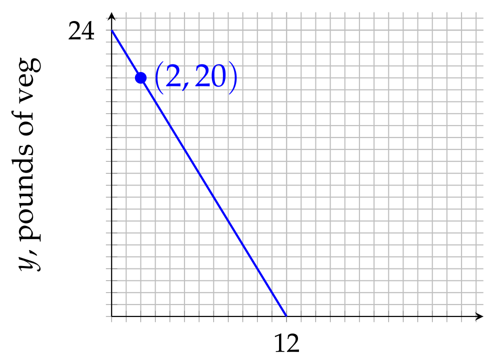
   (a) (6 points) Find an equation relating x and y.

Solution: $2x + y = 24$, or $y = -2x + 24$

1 points	some expression relating x and y
1 points	y is a linear function of x
3 points	substantially correct, maybe a sign error
1 point	correct equation

   (b) (6 points) On the axes below:

   - Graph your equation.
   - Label each axis by writing the name of the variable and its units along the axis.
   - Label the coordinates at the vertical and horizontal intercepts.
   - Draw a dot at the point on your graph corresponding to a purchase of 2 pounds of fruit and label its coordinates.



x, pounds of fruit

Solution:

2 points	correct graph
4 points	1 for each correct bullet point

Interpret "correct graph" as either correctly reflecting the practical description of the problem or correctly matching the expression the student gave in the previous part, to the student's benefit.

4. (6 points) Write the equation of the line perpendicular to $4x + 3y = 9$ that passes through the point $(8, 5)$.

Solution: The slope of $4x + 3y = 9$ is $-\frac{4}{3}$, the perpendicular slope is $\frac{3}{4}$. The perpendicular line's equation is $y = \frac{3}{4}x - 1$ or any equivalent equation.

1 point	some linear function
1 points	uses some slope derived from the given line
1 points	correct slope
2 points	substatially correct equation of line, may be some error
1 point	all correct

5. Let $f(x) = 2x - 7$ and $g(x) = \dfrac{2x-3}{4x+2}$.

(a) (3 points) Evaluate $g(3)$.

Solution: $g(3) = \dfrac{6-3}{12+2} = \dfrac{3}{14}$

1 point	plug in 3 to $g(x)$
1 point	substantially correct evaluation
1 point	correct answer

(b) (3 points) Evaluate $f\big(g(3)\big)$.

Solution: $f\big(g(3)\big) = f\big(\tfrac{3}{14}\big) = 2 \times \tfrac{3}{14} - 7 = \tfrac{3}{7} - 7 = -\tfrac{46}{7} \approx -6.57$

1 points	use correct definition of composition
1 point	substantially correct evaluation
1 point	all correct

Decimal approximations are OK in evaluating function values.

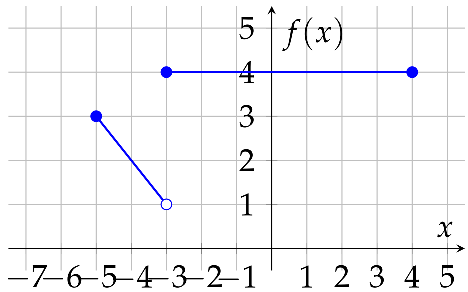
(c) (4 points) Find all values of x solving $g(x) = 3$.

Solution:

$$\begin{aligned} \frac{2x-3}{4x+2} &= 3 \\ 2x - 3 &= 12x + 6 \\ 10x &= -9 \\ x &= -\tfrac{9}{10} \end{aligned}$$

1 point	set $g(x) = 1$
2 points	at least substantially correct algebra
1 point	correct algebra and solution

6. Use the graph of f below to answer the following questions.



(a) (5 points) Fill in the blanks to give a piecewise-defined expression for f.

$$f(x) = \begin{cases} \underline{\quad -x-2 \quad} & \text{for } \underline{-5 \le x < -3} \\ \underline{\quad \mathbf{4} \quad} & \text{for } \underline{-3 \le x \le 4} \end{cases}$$

2 points	non-constant linear part
1 point	constant linear part
2 points	correct subintervals for domain

(b) (3 points) Give the domain and range of f.

- Domain: $\underline{-5 \le x \le 4}$
- Range: $\underline{1 < f(x) \le 3 \textbf{ and } f(x) = 4}$

1 points	correct domain
1 point	at least partially correct range, might be one interval only
1 points	correct correct range

7. The cost (in dollars) of producing x dryers is

$$C = f(x) = 450 + 27x$$

(a) (5 points) Give an expression for $f^{-1}(C)$.

Solution: $f^{-1}(C) = \frac{C-450}{27}$

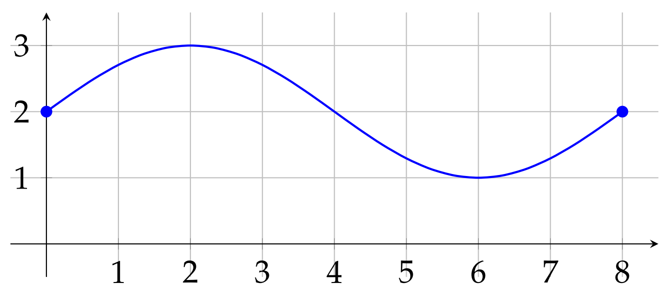
2 points	some evidence of solving $y = C(x)$ for x
2 points	substanially correct expression, may have some mismatched variables
1 point	correct expression with matched variables

By "mismatched variables" I mean something like $f^{-1}(C) = \frac{x-450}{27}$

(b) (3 points) Explain in a sentence the practical meaning of your expression, with correct units.

Solution: $x = f^{-1}(C)$ is the number of dryers produced for a cost of C dollars.

8. (8 points) The graph of $y = f(x)$ is given below.



Give the intervals on which f is simultaneously ...

(a) ...increasing and concave up.

Solution: $6 < x < 8$

(b) ...increasing and concave down.

Solution: $0 < x < 2$

(c) ...decreasing and concave up.

Solution: $4 < x < 6$

(d) ...decreasing and concave down.

Solution: $2 < x < 4$

Solution:	1 point	if given inteval satisfies at least one criterion
	1 point	both criteria satisfied

Any method of notating or describing the intervals is OK.

If you have had specific conversations with your classes you may choose otherwise, but I suggest not distinguishing between $<$ and $\leq$.

9. (8 points) The graph of $y = g(x)$ contains the point $(-6, 18)$. Find a point on the graph of each of the following transformations of g.

   (a) $y = -2g(x)$

   (a) $(-6, -36)$

   (b) $y = g(3x)$

   (b) $(-2, 18)$

   (c) $y = g\left(\frac{1}{2}(x-2)\right) + 2$

   (c) $(-1, 20)$

   (d) $y = \frac{1}{3}g(x) - 2$

   (d) $(-6, 4)$

Solution: For each part:

1 point	at least one correct coordinate
1 points	both coordinates correct

10. A quadratic function passes through $(4, 5)$ and has a vertex at $(6, 2)$.

    (a) (5 points) Give a formula for the quadratic function. Write your formula giving y as a function of x.

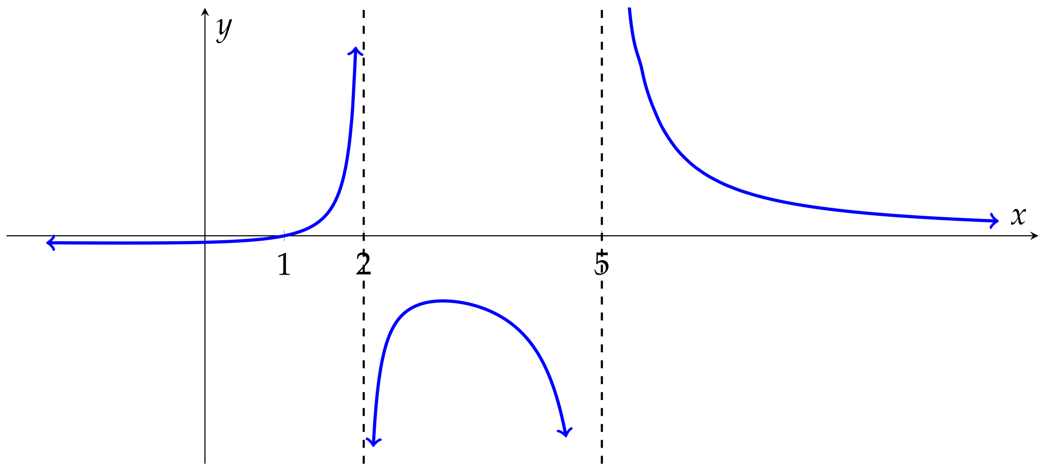
    Solution: $y = \frac{4}{3}(x-6)^2 + 2$

1 point	y as some quadratic function of x
1 point	evidence of using correct vertex form
1 point	correctly using vertex coordinates
1 point	attempt to solve for leading constant
1 point	correct work

    (b) (3 points) Explain in a sentence what aspect of your formula tells you whether this is a concave up or concave down function.

    Solution: Since the leading constant is positive, the function is concave
    (3 points: This seems all-or-nothing.)

11. (8 points) The function f is a rational function with a horizontal asymptote at $y = 0$. Its graph is shown below. Give a possible formula for $f(x)$.



Solution: $f(x) = \dfrac{x-1}{(x-2)(x-5)}$

1 points	some rational function
2 points	correct numerator factor is present
1 point	no incorrect numerator factors
2 points	correct denominator factors present
2 point	no incorrect denominator factors

12. The concentration of a particular mineral in the water in a lake is proportional to the square of the depth. Let $S(x)$ be the mineral concentration at a depth of x feet.

(a) (4 points) Write out the formula for $S(x)$ in terms of x and the constant of proportionality k.

Solution: $S(x) = kx^2$

2 points	some correct proportionality relationship, may use x instead of x^2
2 points	correct expression

(b) (4 points) At a depth of 10 feet, the mineral concentration is 20 grams per liter. Find k and rewrite the formula for S using it.

Solution: Solve $20 = 100k$ to get $k = 0.2$, so $S(x) = 0.2x^2$.

2 points	set up an equation to solve for k based on the formula from part (a)
2 points	correctly solve for k
1 point	write out the correct formula for $S(x)$

(c) (2 points) At what depth is the mineral concentration 180 grams per liter?

Solution: Solve $180 = 0.0x^2$ to get $x = 30$.

1 point	set up $S(x) = 180$ using formula from (b)
1 point	solve for x correctly